Roll No. ........................ **Exam Code : D-22**

# Subject Code—11707

## M. Sc. EXAMINATION

(Batch 2019 Onwards)

(For Regular/Affiliated/Distance Mode)

(First Semester)

MATHEMATICS

MAL-511

Algebra

*Time* : 3 *Hours* *Maximum Marks* : 70

**Note** : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

**1.** (a) Find second commutator subgroup of $D_3$, where $D_3$ is the dihedral group. 2

(b) Write upper and lower central series of a group of order 257. 2

**P.T.O.**

(c) What do you und... series of a group ? Does this series exist for every finite groups ? 2

(d) Give an example of an algebraic extension and a transcendental extension of field of rational numbers. 2

(e) Define normal extension of a field. Give an example of normal extension of the field of rational numbers. 2

(f) Write the cyclotomic polynomial $\phi_{27}(x)$. 2

(g) Write Galois group of the polynomial $x^2 - 2 \in Q(x)$. 2

## Section I

2. (a) State and prove Zassenhaus lemma. 5

(b) Define composition series of a group. Show that each factor of composition series of a finite $p$-group has order $p$. Write all the composition series of $D_4$, where $D_4$ is dihedral group of order 8. 9



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3. (a) State and prove Scheiers refinement theorem. Hence deduce that every two composition series of a group G have same length. 9

(b) If $x$, $y$, $z$ are arbitrary elements of a group then $[x, z, y^x]\,[y, x, z^y]\,[z, y, x^z] = 1$, where 1 is the identity element of G. 5

## Section II

4. (a) Show that a group G is solvable if and only if its $k$th commutator subgroup is identity. Hence deduce that $D_8$ is solvable. 10

(b) Show that $S_5$ is not solvable, where $S_5$ is symmetric group of degree 5. 4

5. (a) Define the upper central series and the lower central series of a group G. If G is a group of class $r$, then show that the length of upper and lower central series of G is also $r$. Discuss the result for a nilpotent group of class three. 9

(b) If H is a proper subgroup of a nilpotent group G, then show that H is a proper subgroup of its normalizer also. Hence deduce that $S_3$ is not nilpotent. 5

## Section III

6. (a) Show that $\alpha$ is algebraic over the field F iff $F(\alpha)$ is a finite extension of F. 7

(b) Show that $\alpha$ and $\beta$ are conjugate over F iff they have same minimal polynomial. 7

7. (a) Show that an extension K of F is a normal extension iff K is the splitting filed of some polynomial over F. 9

(b) Show that a regular n-gon is constructible if $\phi(n)$ is a power of two, where $\phi$ is Euler's Phi function. 5

## Section IV

8. (a) Show that every cyclotomic polynomial $\phi_n(x)$ is irreducible over Q. 6

(b) Find the Galois Groups of polynomials $x^4 - 2$ and $x^4 - 3x^2 +$ over the field of rational numbers

9. (a) If K is a normal extension of F and T a subfield of K containing F, then sh that T is normal extension of F G (K, T) is a normal subgroup G(K, F).

(b) Let $f(x)$ be solvable by radicals over Show that the Galois group is solvab